

## Electronic Supplementary Material

Here we provide meridional plots of the components of the three Dudley-James type flows ( $t_1^0 s_1^0$ ,  $t_1^0 s_2^0$  and  $t_2^0 s_2^0$ ), in order to give more detail about the structure of these optimal flows.

### $t_1^0 s_1^0$ optimum

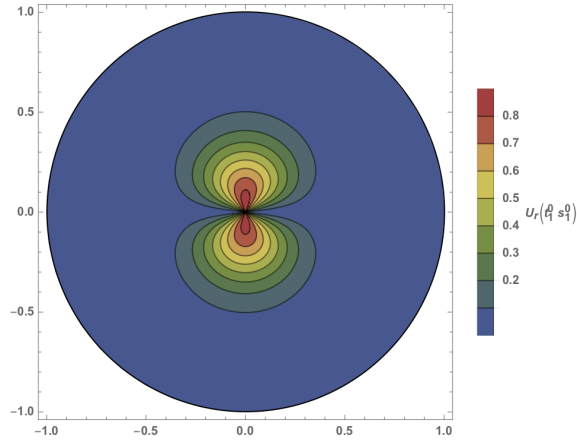


Figure 1: Meridional plot of the radial component of the  $t_1^0 s_1^0$  optimal flow.

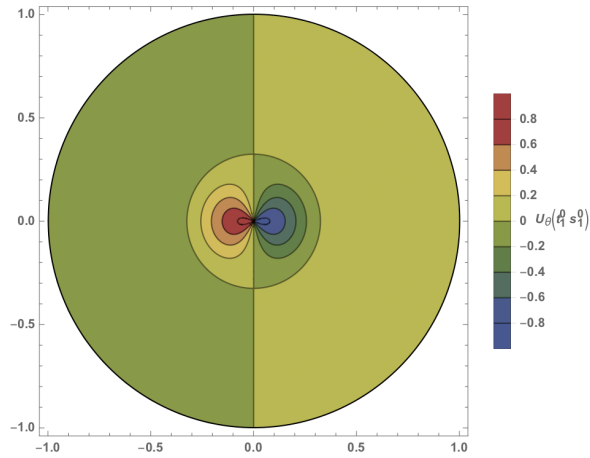


Figure 2: Meridional plot of the theta component of the  $t_1^0 s_1^0$  optimal flow.

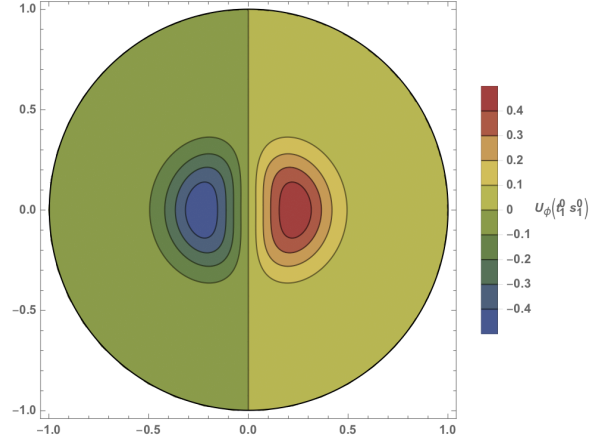


Figure 3: Meridional plot of the  $\phi$  component of the  $t_1^0 s_1^0$  optimal flow.

$t_1^0 s_2^0$  optimum

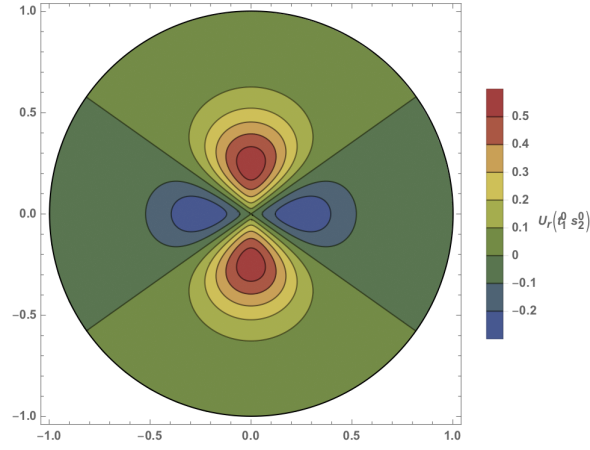


Figure 4: Meridional plot of the radial component of the  $t_1^0 s_2^0$  optimal flow.

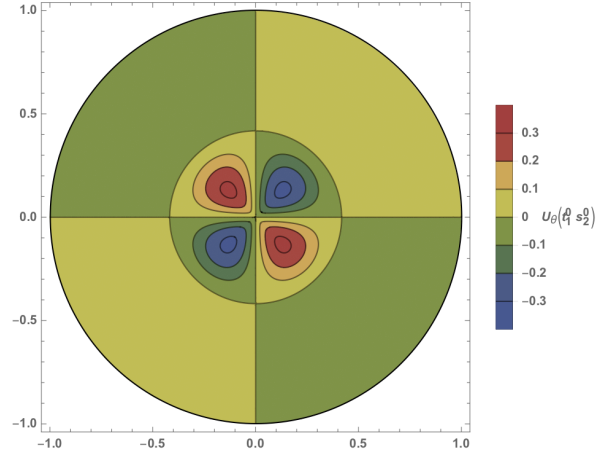


Figure 5: Meridional plot of the theta component of the  $t_1^0 s_2^0$  optimal flow.

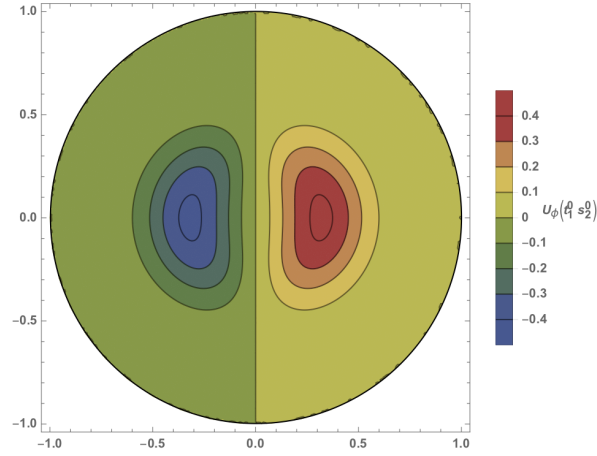


Figure 6: Meridional plot of the phi component of the  $t_1^0 s_2^0$  optimal flow.

### $t_2^0 s_2^0$ optimum

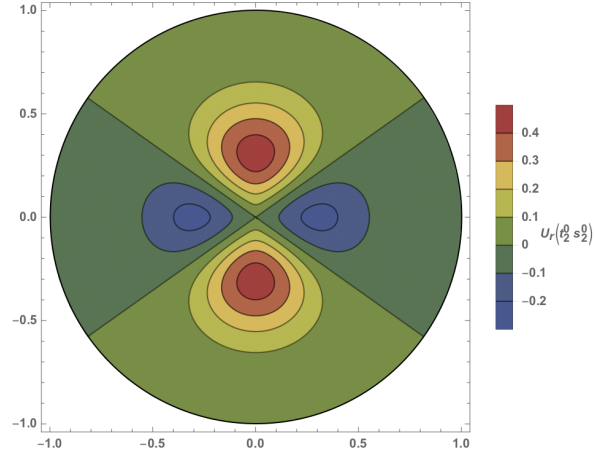


Figure 7: Meridional plot of the radial component of the  $t_2^0 s_2^0$  optimal flow.

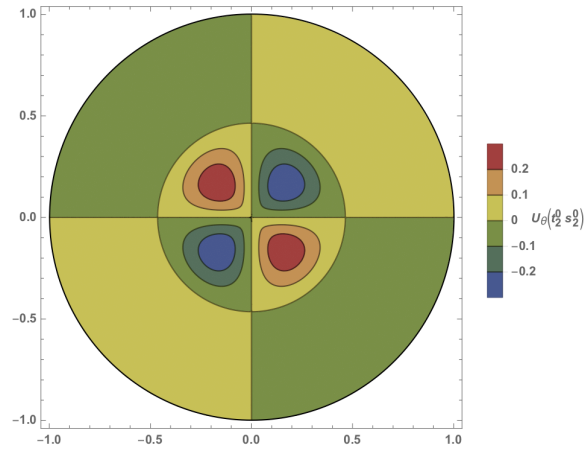


Figure 8: Meridional plot of the theta component of the  $t_2^0 s_2^0$  optimal flow.

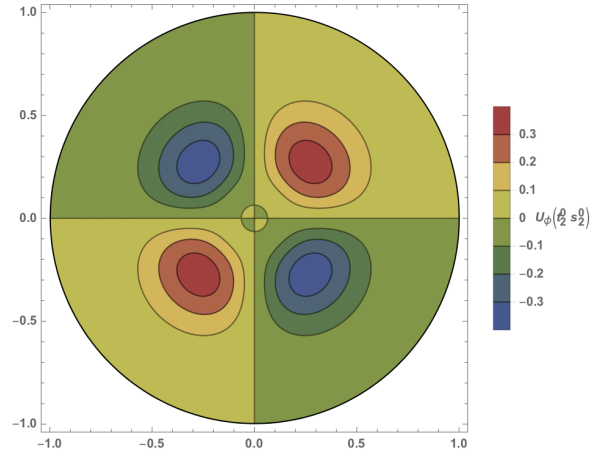


Figure 9: Meridional plot of the phi component of the  $t_2^0 s_2^0$  optimal flow.